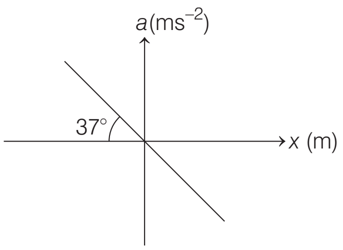
Unit Test 2
(Waves and Oscillations)

1 A mass m attached to a spring of spring constant k is stretched a distance x_0 from its equilibrium position and released with no initial velocity. The maximum speed attained by the mass in its subsequent motion and the time at which this speed would be attained are respectively,

(a) $\sqrt{\frac{k}{m}}x_0, \pi\sqrt{\frac{m}{k}}$ (b) $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \frac{\pi}{2}\sqrt{\frac{m}{k}}$

(c) $\sqrt{\frac{k}{m}}x_0, \frac{\pi}{2}\sqrt{\frac{m}{k}}$ (d) $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \pi\sqrt{\frac{m}{k}}$

2 The acceleration-displacement graph of a particle executing SHM is shown in the figure. The time period of SHM is



(a) $\frac{4\pi}{\sqrt{3}}$s

(b) $\frac{2\pi}{\sqrt{3}}$s

(c) The given graph doesn't represent SHM

(d) Information is insufficient

3 A spring balance has a scale that can read from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates harmonically with a time period of 0.6s. The mass of the body is (Take, $g = 10\text{ms}^{-2}$)

(a) 10 kg (b) 25 kg
(c) 18 kg (d) 22.8 kg

4 A block is kept on a table which performs simple harmonic motion with frequency 5Hz in horizontal plane. The maximum amplitude of the table at which block does not slip on the surface of table is, (if coefficient of friction between the block and surface of table is 0.6.) (Given, $g = 10$ m/s^2).

(a) 0.06 m (b) 0.006 m
(c) 0.02 m (d) 0.002 m

5 For a particle executing SHM, determine the ratio of average acceleration of the particle between extreme position and the equilibrium position w.r.t. the maximum acceleration

(a) $\frac{4}{\pi}$ (b) $\frac{2}{\pi}$
(c) $\frac{1}{\pi}$ (d) $\frac{1}{2\pi}$

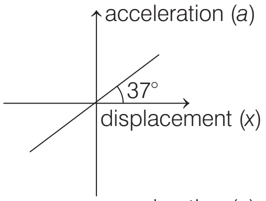
6 Two springs are made to oscillate simple harmonically when the same mass is suspended, individually. The time periods obtained are T_1 and T_2. If both the springs are connected in series and then made to oscillate when suspended by the same mass, the resulting time will be

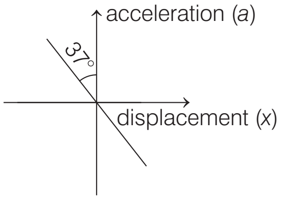
(a) $T_1 + T_2$ (b) $\frac{T_1T_2}{T_1 + T_2}$
(c) $\sqrt{T_1^2 + T_2^2}$ (d) $\frac{T_1 + T_2}{2}$

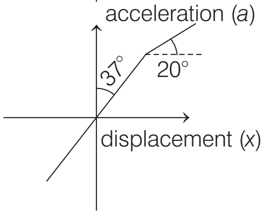
7 Find the time period of oscillations of a torsional pendulum, if the torsional constant of wire is $10\pi^2$ in SI units. The moment of inertia of the rigid body is 10 kg - m^2 about the axis of rotation

(a) 1 s (b) 2 s
(c) 4 s (d) $\frac{1}{2}$ s

8 Acceleration-displacement graph for four particles are shown, identify the one which represents SHM for all the values of displacements

(a) 

(b) 

(c) 

(d) None of these

9 In case of simple harmonic motion, if the velocity is plotted along the X-axis and the displacement from the equilibrium position is plotted along the Y-axis, the resultant curve happens to be an ellipse with the ratio $\frac{\text{major axis (along } X)}{\text{minor axis (along } Y)} = 20\pi$.

What is the frequency of the simple harmonic motion?

(a) 100 Hz (b) 20 Hz
(c) 10 Hz (d) $\frac{1}{10}$ Hz

10 A simple pendulum is suspended from the ceiling of a stationary tram car. Now, the car starts accelerating, the time period of a simple pendulum is the least when [Take magnitude of acceleration to be same in all the cases]

(a) car is accelerating up
(b) car is accelerating down
(c) car is accelerating horizontally
(d) car is stationary

11 A particle executes SHM about O with an amplitude A and time period T. The magnitude of its acceleration, at $\frac{T}{8}$ s after the particle reaches the extreme position, would be

(a) $\frac{4\pi^2 A}{\sqrt{2}T^2}$ (b) $\frac{4\pi^2 A}{T^2}$
(c) $\frac{2\pi^2 A}{\sqrt{2}T^2}$ (d) None of these

12 A string of length 1.5 m with its two ends clamped, is vibrating in the fundamental mode. The amplitude at the centre of the string is 4 mm. The minimum distance between two points having amplitude 2 mm, is

(a) 1 m (b) 75 cm
(c) 60 cm (d) 50 cm

13 A spring of negligible mass having a force constant k extends by an amount y when a mass m is hung from it. The mass is pulled down a little and then released. The system begins to execute SHM of amplitude A and angular frequency ω. The total energy of the mass-spring system will be

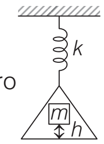
(a) $\frac{m\omega^2 A^2}{2}$ (b) $\frac{m\omega^2 A^2}{2} - \frac{ky^2}{2}$
(c) $\frac{ky^2}{2}$ (d) $\frac{m\omega^2 A^2}{2} + \frac{ky^2}{2}$

14 Total number of independent harmonic waves in the resultant displacement equation given by

$$y = 3\sin^2\frac{t}{2}\sin 800t$$

(a) 2 (b) 1 (c) 4 (d) 3

15 A load of mass m falls from a height h on the scale pan hung from a spring as shown. If the spring constant is k, mass of the scale pan is zero and the mass m does not bounce relative to the pan, then the amplitude of vibration is



(a) $\frac{mg}{k}$ (b) $\frac{mg}{k}\sqrt{1+\frac{2hk}{mg}}$
(c) $\frac{mg}{k}+\frac{mg}{k}\sqrt{\frac{1+2hk}{mg}}$ (d) None of these

16 A sound absorber attenuates the sound level by 30dB. The intensity of sound is decreased by a factor of

(a) 100 (b) 1000
(c) 10000 (d) 10

17 The velocity of a particle executing a simple harmonic motion is 13 ms^{-1}, when its distance from the equilibrium position is 3 m and its velocity is 12 ms^{-1}, when it is 5 m away from equilibrium position. The frequency of the SHM is

(a) $\frac{5\pi}{8}$ (b) $\frac{5}{8\pi}$
(c) $\frac{8\pi}{5}$ (d) $\frac{8}{5\pi}$

18 Two identical strings A and B, have nearly the same tension. When they both vibrate in their fundamental resonant modes, there is a beat frequency of 3 Hz. When string B is tightened slightly, to increase the tension, the beat frequency becomes 6 Hz. This means

(a) that before tightening A had a higher frequency than B, but after tightening, B has a higher frequency than A
(b) that before tightening B has higher frequency than A, but after tightening A has higher frequency than B
(c) that before and after tightening A has higher frequency than B
(d) that before and after tightening B has higher frequency than A

19 The ratio of densities of oxygen and nitrogen is 16 : 14. At what temperature, the speed of sound in oxygen will be equal to its speed in nitrogen at 14°C?

(a) 50°C (b) 52°C
(c) 48°C (d) 55°C

20 A train is passing by a platform at a constant speed of 40 ms^{-1}. The horn of the train has a frequency of 320 Hz. Find the overall change in frequency detected by a person standing on the platform, i.e. when the train approaching and then precedes from him. (Take, velocity of sound in air as 320 ms^{-1})

(a) 216.4 Hz (b) 81.3 Hz
(c) 365.7 Hz (d) 284.4 Hz

21 A string of length 0.4 m and mass 10^{-2} kg is clamped at one end. The tension in the string is 1.6 N. Identical wave pulses are generated at the free end, after a time interval Δt. The minimum value of Δt, so that a constructive interference takes place between successive pulses is

(a) 0.1s
(b) 0.05 s
(c) 0.2 s
(d) Constructive interference cannot take place

22 A string vibrates according to the equation

$Y = 5 \sin\left(\frac{2\pi x}{3}\right) \times \cos 20\pi t$, where x and y are in cm and t in second. The distance between two adjacent nodes is

(a) 3 cm (b) 4.5 cm (c) 6 cm (d) 1.5 cm

23 A point source of sound is placed in a non-absorbing medium. Two points A and B are at the distance of 1 m and 2 m, respectively from the source. The ratio of amplitudes of waves at A to B is

(a) 1 : 1 (b) 1 : 4 (c) 1 : 2 (d) 2 : 1

24 The mathematical form of three travelling waves are given by

$y_1 = (2\text{ cm}) \sin(3x - 6t)$,

$y_2 = (3\text{ cm}) \sin(4x - 12t)$,

and $y_3 = (4\text{ cm}) \sin(5x - 11t)$

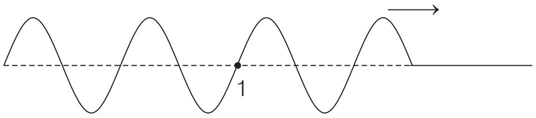
of these waves,

(a) wave 1 has greatest wave speed and wave has maximum transverse string speed
(b) wave 2 has greatest wave speed and wave 1 has greatest maximum transverse string speed
(c) wave 3 has greatest wave speed and wave 1 has maximum transverse string speed
(d) wave 2 has greatest wave speed and wave 3 has maximum transverse string speed

25 If the maximum speed of a particle carrying a travelling wave is v_0, then find the speed of a particle when the displacement is half that of the maximum value

(a) $\frac{v_0}{2}$ (b) $\frac{\sqrt{3}v_0}{4}$ (c) $\frac{\sqrt{3}v_0}{2}$ (d) v_0

26 A transverse wave on a string travelling along positive x-axis has been shown in the figure below



The mathematical form of the wave is shown

$$y = (3.0\text{ cm}) \sin\left[2\pi \times 0.1t - \frac{2\pi}{100}x\right]$$

where t is in seconds and x is in cm. Find total distance travelled by the particle at the (1), in 10 min 15 s, measured from the instant shown in the figure and direction of the motion of the particle at the end of this time.

(a) 6 cm, in upward direction
(b) 6 cm, in downward direction
(c) 738 cm, in upward direction
(d) 732 cm, in upward direction

27 A wire having a linear mass density of 5×10^{-3} kg m^{-1} is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490Hz. The length of the wire is

(a) 2.5 m (b) 2.14 m
(c) 2.25 m (d) 2.0 m

28 A man stands on a weighing machine placed on a horizontal plateform. The machine reads 50 kg. By means of a suitable mechanism, the plateform is made to execute harmonic vibration up and down with a frequency of 2 vibrations per second. What will be the effect on the reading of the weighing machine? The amplitude of vibration of plateform is 5 cm. (Take, $g = 10\ ms^{-2}$)

(a) 11 kgf to 93 kgf
(b) 10.5 kgf to 89.5 kgf
(c) 10 kgf to 15.5 kgf
(d) 25.6 kgf to 100.5 kgf

29 A small trolley of mass 2.0 kg resting on a horizontal turn table is connected by a light spring to the centre of the table. When the turn table is set into rotation at speed of 360 rpm, the length of the stretched spring is 43 cm. If the original length of the spring is 36 cm, the force constant is

(a) 17025 Nm^{-1} (b) 16225 Nm^{-1}
(c) 17475 Nm^{-1} (d) 17555 Nm^{-1}

30 At 16°C, two open end organ pipes, when sounded together give 34 beats in 2 s. How many beats per second will be produced, if the temperature is raised to 51°C?

(Neglect increase in length of the pipes)

(a) 18 s^{-1} (b) 15 s^{-1}
(c) 20 s^{-1} (d) 10 s^{-1}

31 During earthquake, both longitudinal and transverse waves are produced having speeds 4.0 km/s and 8.0 km/s, respectively. If the first transverse wave reaches the seismograph 8 minutes after the arrival of first longitudinal wave, then the distance of the position, where the earthquake occurred is

(a) 3440 km (b) 3880 km
(c) 3840 km (d) 3500 km

32 A pendulum has time period T in air. When it is made to oscillate in water, it acquired a time period $T' = \sqrt{2}T$. The density of the pendulum bob is equal to
(Take, density of water = 1)

(a) $\sqrt{2}$ (b) 2
(c) $2\sqrt{2}$ (d) None of these

33 On a planet a freely falling body takes 2 s when it is dropped from a height of 8 m. The time period of simple pendulum of length 1 m on that planet is

(a) 3.14 s (b) 16.28 s
(c) 1.57 s (d) None of these

Direction (Q. Nos. 34-40) *Each of these questions contains two statements Statement I and Statement II. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

34 **Statement I** Waves on a string can be longitudinal in nature.

Statement II The string cannot be compressed or rarified.

35 **Statement I** A wave of frequency 500 Hz is propagating with a velocity of 350 m/s. Distance between two particles with 60° phase difference is 12 cm.

Statement II $x = \frac{\lambda}{2\pi}\phi$

36 **Statement I** When a wave goes from one medium to other, average power transmitted by the wave may change.

Statement II Due to a change in the medium, amplitude, speed, wavelength and frequency of the wave may change.

37 **Statement I** A particle performs a simple harmonic motion with amplitude A and angular frequency ω. To change the angular frequency of simple harmonic motion to 3ω, and amplitude to $A/2$, we have to supply an extra energy of $\frac{5}{4}m\omega^2 A^2$, where m is the mass of the particle executing simple harmonic motion.

Statement II Angular frequency of the simple harmonic motion is independent of the amplitude of oscillation.

38 **Statement I** Time period of spring pendulum is the same whether in an accelerated or in an inertial frame of reference.

Statement II Mass of the bob of the spring pendulum and the spring constant of spring are independent of the acceleration of the frame of reference.

39 **Statement I** The total energy of a particle executing simple harmonic motion, can be negative.

Statement II Potential energy of a system can be negative.

40 **Statement I** A circular metal hoop is suspended on the edge, by a hook. The hoop can oscillate from one side to the other in the plane of the hoop, or it can oscillate back and forth in a direction perpendicular to the plane of the hoop. The time period of oscillation would be more when oscillations are carried out in the plane of hoop.

Statement II Time period of physical pendulum is more, if the moment of inertia of the rigid body about the corresponding axis passing through the pivoted point, is more.

ANSWERS

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (c)
11. (a)	**12.** (a)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (d)	**20.** (b)
21. (c)	**22.** (a)	**23.** (d)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (a)
31. (c)	**32.** (b)	**33.** (a)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (a)

Hints and Explanations

1 At the mean position, the speed will be maximum.

$$\frac{kx_0^2}{2} = \frac{mv^2}{2}$$

$$\Rightarrow \quad v = v_{\max} = \sqrt{\frac{k}{m}}\, x_0$$

and this is attained at $t = \frac{T}{4}$.

Time period of motion is,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

So, the required time is,

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

2 $\frac{da}{dx} = -\tan 37° = -\frac{3}{4} \Rightarrow a = -\frac{3}{4}x$

On comparing with $a = -\omega^2 x$, we get

$$\omega^2 = \frac{3}{4} \Rightarrow \frac{2\pi}{T} = \frac{\sqrt{3}}{2} \Rightarrow T = \frac{4\pi}{\sqrt{3}}\text{ s}$$

3 The scale can read a maximum of 50 kg, for a length of 20 cm. Let spring constant be k then,

$$kx_0 = mg$$

[for $m = 50$ kg, $x_0 = 20$ cm]

$\Rightarrow k \times 0.2 = 50 \times 10 \Rightarrow k = 2500$ Nm

Let mass of the body be m_0, then from

$$T = 2\pi\sqrt{\frac{m_0}{k}} \Rightarrow 0.6 = 2\pi\sqrt{\frac{m_0}{2500}}$$

$$\Rightarrow \quad m_0 = 22.8 \text{ kg}$$

4 Maximum force on the block on the surface of table due to simple harmonic motion, $F = m\omega^2 A$, where $A \rightarrow$ amplitude.

Friction force on the block, $F_s = \mu mg$

It will not slip on the surface of the table, if

$$F = F_s$$

$$m\omega^2 A = \mu mg$$

$$A = \frac{\mu g}{\omega^2}$$

$$= \frac{0.6 \times 10}{(2 \times 3.14 \times 5)^2}$$

$$= 0.006 \text{ m}$$

5 Let the equation of SHM be,

$x = A \sin \omega t$

Average acceleration between extreme position and the equilibrium position, i.e. from time $t = 0$ to $t = \frac{T}{4}$

$$I_m = \frac{\int_0^{T/4} \omega^2 A \sin \omega t \, dt}{T/4}$$

Maximum acceleration $(a_{\max}) = \omega^2 A$.

Then, the required ratio is,

$$= \frac{\int_0^{T/4} \omega^2 A \sin \omega t \, dt}{T/4 \times \omega^2 A} = \frac{2}{\pi}$$

6 Let the spring constants of the two springs be k_1 and k_2 respectively, then,

$$T = 2\pi\sqrt{\frac{m}{K}} \quad \text{and } k = \frac{4\pi^2 m}{T^2}$$

When the two springs are connected in series, then

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eq}}}}$$

where, $\quad k_{\text{eq}} = \frac{k_1 k_2}{k_1 + k_2}$

$$\Rightarrow \quad T = \sqrt{T_1^2 + T_2^2}$$

7 For torisional pendulum, $\tau = -k\theta$

$$\alpha = -\frac{k}{I}\theta$$

$$\Rightarrow \quad \omega^2 = \frac{k}{I}$$

$$T = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{10}{10\pi^2}} = 2\text{ s}$$

8 For a particle to execute SHM,

$a = -\omega^2 x$

So, $\frac{da}{dx} = -\omega^2$, where ω^2 is positive quantity. This means for a particle to execute SHM, the acceleration-displacement curve should be a straight line having a negative slope, which is shown in option (b).

9 In representation of SHM in ellipse as velocity along x-axis and displacement along y-axis.

$$\because \frac{\text{Major axis (along } X\text{-axis)}}{\text{Minor axis (along } Y\text{-axis)}} = 20\pi$$

$$\Rightarrow \frac{\omega A}{A} = 20\pi \Rightarrow \omega = 20\pi$$

$\Rightarrow 2\pi f = 20\pi$

The frequency of SHM, $f = 10$ Hz

10 When car is accelerating horizontally

$$g_{\text{eff}} = \sqrt{g^2 + a^2}$$

11 Let at $t = 0$, the particle be at the extreme position, then the equation of SHM can be written as

$$x = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T} \times t\right)$$

At $t = \frac{T}{8}$, $x = A\cos\frac{\pi}{4} = \frac{A}{\sqrt{2}}$

$$\text{Acceleration} = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 \times \frac{A}{\sqrt{2}}$$

$$\text{Magnitude of acceleration} = \frac{4\pi^2 A}{\sqrt{2}\,T^2}$$

12 $A_s = 2A \sin kx$

$$2\text{ mm} = 4\text{ mm} \sin kx$$

$$\Rightarrow \quad kx = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\Rightarrow \quad x_2 - x_1 = \left[\frac{5\pi}{6} - \frac{\pi}{6}\right] \times \frac{1}{k} = \frac{\lambda}{3}$$

As the string is vibrating in fundamental mode

$$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L = 3\text{ m}$$

So, required separation between two points,

$$x_2 - x_1 = 1\text{ m}$$

13 From initial equilibrium position,

$$ky = mg$$

When block is at distance x below mean position

Kinetic energy of the block,

$$K = \frac{m\omega^2 A^2}{2}\cos^2(\omega t - \phi)$$

[From SHM theory]

Elastic potential energy of spring-block-earth system,

$$U_e = \frac{k(y + x)^2}{2}$$

where, $x = A\sin(\omega t + \phi)$

Gravitational potential energy of spring-block-earth system is, $U_g = -mgx$

Taking, mean position as reference position for gravitation potential energy.

Total energy,

$$E = K + U_e + U_g = \frac{m\omega^2 A^2}{2} + \frac{ky^2}{2}$$

14 $y = 3\sin^2 \frac{t}{2} \sin 800t$

$$= 3\left(\frac{1 - \cos t}{2}\right)\sin 800t$$

$$= \frac{3}{2}\sin 800t - \frac{3}{2}\sin 800t \cos t$$

$$= \frac{3}{2}\sin 800t - \frac{3}{4}[2\sin 800t \cos t]$$

$$= \frac{3}{2}\sin 800t - \frac{3}{4}[\sin 801t + \sin 799t]$$

$$= \frac{3}{2}\sin 800t - \frac{3}{4}\sin 801t - \frac{3}{4}\sin 799t$$

$\therefore$ Number of independent harmonic wave $= 3$.

15 From the conservation principle,

$$mgh = \frac{1}{2}kx_0^2 - mgx_0$$

where, x_0 is maximum elongation in spring.

$$\Rightarrow \frac{1}{2}kx_0^2 - mgx_0 - mgh = 0$$

$$\Rightarrow x_0^2 - \frac{2mg}{k}x_0 - \frac{2mg}{k}h = 0$$

$$\Rightarrow x_0 = \frac{2\frac{mg}{2} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + 4 \times \frac{2mg}{k}h}}{2}$$

Amplitude = Elongation for lowest extreme position – elongation for equilibrium position.

$$= x_0 - x_1 = \frac{mg}{k}\sqrt{1 + \frac{2hk}{mg}}\left[\because x_1 = \frac{mg}{k}\right].$$

16 If I_1 be initial intensity of sound and I_2 be the final intensity of sound,

then $S_1 = 10\log\frac{I_1}{I_0}$ and $S_2 = 10\log\frac{I_2}{I_0}$

$$S_1 - S_2 = 30 \text{ dB}$$

$$10\log\frac{I_1}{I_0} - 10\log\frac{I_2}{I_0} = 30$$

$$10\log\left(\frac{I_1}{I_0}/\frac{I_2}{I_0}\right) = 30$$

$$10\log\frac{I_1}{I_2} = 30$$

$$\log\frac{I_1}{I_2} = 3$$

$$\frac{I_1}{I_2} = 10^3$$

$$I_2 = \frac{1}{1000}I_1$$

17 Speed in SHM, $v = \omega\sqrt{a^2 - x^2}$

$$\Rightarrow v^2 = \omega^2(a^2 - x^2)$$

According to question,

$$v_1^2 = \omega^2(a^2 - x_1^2)$$

$$v_2^2 = \omega^2(a^2 - x_2^2)$$

$$\Rightarrow v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$$

$$\Rightarrow \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} = \sqrt{\frac{(13)^2 - (12)^2}{(5)^2 - (3)^2}}$$

$$= \sqrt{\frac{169 - 144}{25 - 9}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Also, $\omega = 2\pi f = \frac{5}{4} \Rightarrow f = \frac{5}{8\pi}$.

18 Let the fundamental frequencies of A and B, before tightening of B are f_1 and f_2, respectively.

Then either $f_1 - f_2 = 3$ or $f_2 - f_1 = 3$.

As tension in B increases (due to tightening), its frequency increases and the beat frequency also increases.

If $f_1 - f_2 = 3$, then according to given condition, when f_2 increases $f_1 - f_2$ the decreases so the frequencies of strings are related by $f_2 - f_1 = 3$.

i.e. before tightening, $f_2 > f_1$

After tightening,

$$f_2' - f_1 = 6,$$

i.e. $$f_2' > f_1$$

19 Speed of sound, $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

Speed of sound in oxygen at t°C,

$$v_{oxy} = \sqrt{\frac{\gamma R(t + 273)}{M_{oxy}}}$$

Speed of nitrogen at 14°C,

$$v_N = \sqrt{\frac{\gamma R(14 + 273)}{M_N}}$$

As $$v_{oxy} = v_N$$

$$\Rightarrow \frac{M_N}{M_{oxy}} = \frac{14 + 273}{t + 273}$$

$$\Rightarrow \frac{\rho_N}{\rho_{oxy}} = \frac{14 + 273}{t + 273}$$

$$\Rightarrow \frac{14}{16} = \frac{14 + 273}{t + 273}$$

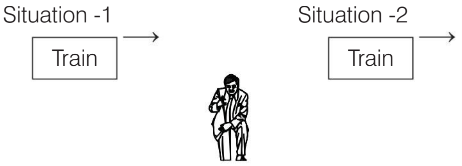
$$\Rightarrow t = \frac{2296 - 1911}{7} = 55°\text{ C}.$$

20 For situation 1,

$$f_{ap1} = \frac{v - 0}{v - v_s} \times f$$

$$= \frac{320}{320 - 40} \times 320$$

$$= 365.7 \text{ Hz}$$



For situation 2,

$$f_{ap2} = \frac{v}{v - (-v_s)} \times f$$

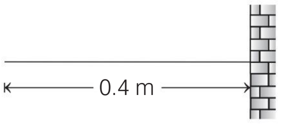
$$= 284.4 \text{ Hz}$$

$$\Delta f = f_{ap1} - f_{ap2}$$

$$= 81.3 \text{ Hz}$$

21 Velocity of wave on the string

$$= \sqrt{\frac{T}{\mu}} = 8 \text{ ms}^{-1}$$



The pulse gets inverted after reflection from the fixed end, so for constructive interference to take place between successive pulses, the first pulse has to undergo two reflections from the fixed end.

So, $\Delta t = \frac{2 \times 0.4 + 2 \times 0.4}{8} = 0.2$ s

22 The node and antinodes are formed in a standing wave pattern as a result of the interference of two waves.

Distance between two nodes is half of wavelength (λ).

So, standard equation of standing wave is

$$y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda} \quad \ldots\text{(i)}$$

where, a is amplitude, λ is wavelength, v is velocity and t is time.

Given equation

$$y = 5\sin\frac{2\pi x}{3}\cos 20\pi t \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{2\pi x}{\lambda} = \frac{2\pi x}{3}$$

$$\Rightarrow \lambda = 3 \text{ cm}$$

23 Let the power of source be P and let it be placed at Q.

Then, intensity at A and at B would be given by

$$I_A = \frac{P}{4\pi \times 1^2}$$

and $$I_B = \frac{P}{4\pi \times 2^2}$$

$$\Rightarrow \frac{(\text{Amp.})_A}{(\text{Amp.})_B} = \sqrt{\frac{I_A}{I_B}} = \sqrt{\frac{2^2}{1^2}} = 2:1$$

24 For the wave, $y = A\sin(kx - \omega t)$,

the wave speed is $\frac{\omega}{k}$ and the maximum transverse string speed is $A\omega$.

Hence option (d) is correct.

25 For the wave $y = A\sin(\omega t - kx)$,

$$v_0 = A\omega$$

where A is the maximum displacement.

For the given condition,

$$\frac{A}{2} = A\sin(\omega t - kx)$$

$$\Rightarrow \sin(\omega t - kx) = \frac{1}{2}$$

and $$\frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$$

$$= A\omega\frac{\sqrt{3}}{2} = \frac{\sqrt{3}v_0}{2}$$

26 We have, $T = \frac{1}{0.1}$ s = 10 s

In one complete cycle, particle travels a distance, 4 times the amplitude.

So, in a time interval of 10 min 15 s i.e., 615 s i.e., 61 full + 1 half- cycles, the distance travelled

$$= (4 \times 3) \times 61 + (2 \times 3) \times 1$$
$$= 732 + 6 = 738 \text{ cm}$$

At this instant, the particle is moving in an upward direction.

27 The frequency of vibration is

$$\nu = \frac{\rho}{2l}\sqrt{\frac{T}{m}}$$

As $\quad 420 = \frac{\rho}{2l}\sqrt{\frac{T}{m}}$

and $\quad 490 = \frac{\rho+1}{2l}\sqrt{\frac{T}{m}}$

$$\Rightarrow \frac{420}{490} = \frac{\rho}{\rho+1} = \frac{6}{7}$$

$$\Rightarrow 7\rho = 6\rho + 6 \Rightarrow \rho = 6$$

$$\therefore 420 = \frac{6}{2l}\sqrt{\frac{450}{5\times10^{-3}}}$$

$$\Rightarrow l = \frac{900}{420} = 2.14 \text{ m}$$

28 Maximum acceleration,

$$A_{max} = \omega^2 a = (2\pi\nu)^2 a = 4\pi^2\nu^2 a$$
$$= 4 \times \left(\frac{22}{7}\right)^2 \times (2)^2 + 0\cdot05$$
$$= 7.9 \text{ ms}^{-2}$$

Maximum force on the man

$$= m(g + A_{max})$$
$$= 50(10 + 7.9) = 895\text{N}$$
$$= 89.5 \text{ kgf}$$

Minimum force on the man

$$= m(g - A_{max})$$
$$= 50(10 - 7.9) = 105\text{N} = 10.5 \text{ kgf}$$

Hence, the reading of weighing machine varies between 10.5 kgf and 89.5 kgf.

29 Here $m = 2$ kg; $\nu = \frac{360}{60} = 6$ rps

Extension produced in spring,

$$y = 43 - 36 = 7 \text{ cm}$$
$$= 7 \times 10^{-2} \text{ m}$$

On rotation the required centripetal force is provided by tension in spring i.e. $ky = mr(2\pi\nu)^2 = 4\pi^2\nu^2 mr$

$$\Rightarrow k = \frac{4\pi^2\nu^2 mr}{y}$$

where, r is length of stretched spring.

$$\Rightarrow k = \frac{4\times(22/7)^2\times6^2\times2\times(43\times10^{-2})}{7\times10^{-2}}$$
$$= 17475 \text{ Nm}^{-1}.$$

30 Let l_1 and l_2 be the lengths of the two pipes, then

$$m = n_1 - n_2 = \frac{\nu_{16}}{2l_1} - \frac{\nu_{16}}{2l_2}$$

$$\Rightarrow \nu_{16}\left(\frac{1}{2l_1} - \frac{1}{2l_2}\right) = 17 \quad \text{... (i)}$$

and $\nu_{51}\left(\frac{1}{2l_1} - \frac{1}{2l_2}\right) = m \quad \text{... (ii)}$

Divide Eq. (ii) by Eq. (i), we get

$$\frac{\nu_{51}}{\nu_{16}} = \frac{m}{17} = \sqrt{\frac{273+51}{273+16}} = \sqrt{\frac{324}{289}}$$

$$\Rightarrow m = \frac{18}{17} \times 17 = 18 \text{ s}^{-1}$$

31 Let d be the distance, then time taken by longitudinal wave,

$$t_1 = \frac{d}{v_1} = \frac{d}{4} \text{ s}$$

Time taken by transverse wave,

$$t_2 = \frac{d}{v_2} = \frac{d}{8} \text{s}$$

As, $t_1 - t_2 = 8 \text{ min} = 8 \times 60$ s

$$\therefore \frac{d}{4} - \frac{d}{8} = 480 \Rightarrow \frac{d}{8} = 480$$

$$\Rightarrow d = 480 \times 8 = 3840 \text{ km}.$$

32 The effective acceleration of bob in water

$$g' = g\left(1 - \frac{\sigma}{\rho}\right)$$

where, σ = density of water and ρ = density of bob

The time period of bob in air, $T = 2\pi\sqrt{\frac{l}{g}}$

The time period in water, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$\therefore \frac{T}{T'} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{g\left(1-\frac{\sigma}{\rho}\right)}{g}}$$
$$= \sqrt{1 - \frac{\sigma}{\rho}} = \sqrt{1 - \frac{1}{\rho}}$$

Given that, $\frac{T}{T'} = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{2} = 1 - \frac{1}{\rho}$

or $\quad \rho = 2$

33 The body is dropped from a height h and takes time t, then

$$h = ut + \frac{1}{2}g_p t^2$$

$$h = \frac{1}{2}g_p t^2 \quad (\because u = 0)$$

$$g_p = \frac{2h}{t^2} = \frac{2\times8}{4} = 4 \text{ ms}^{-2}$$

Time period, $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{4}} = \frac{2\pi}{2}$

$$= \pi = 3.14 \text{ s}$$

34 For longitudinal wave, the medium has to compress and rarify while the string cannot be compressed or rarified.

35 $\lambda = \frac{v}{n} = \frac{350}{500} = 0.7$ m

$$\phi = 60° = 60 \times \frac{\pi}{180°} = \frac{\pi}{3} \text{ radian}$$

As, $x = \frac{\lambda}{2\pi}\phi$

$$\Rightarrow x = \frac{0.7}{2\pi} \times \left[\frac{\pi}{3}\right] = 0.12 \text{ m} = 12 \text{ cm}$$

36 $P_{av} = \frac{\rho v\omega^2 A^2}{2}$

37 Total initial energy of particle in SHM,

$$E_1 = \frac{1}{2}m\omega^2 A^2$$

Energy when amplitude is $\frac{A}{2}$ and angular frequency is 3ω.

$$E_2 = \frac{1}{2}m(3\omega)^2\left(\frac{A}{2}\right)^2 = \frac{9}{8}m\omega^2 A^2$$

$\therefore$ Extra energy $= E_2 - E_1$

$$= \frac{9}{8}m\omega^2A^2 - \frac{1}{2}m\omega^2A^2$$
$$= \frac{5}{8}m\omega^2A^2 = \frac{5}{4}E_1 \text{(gain)}$$

38 The time period of a spring pendulum is given by, $T = 2\pi\sqrt{\frac{m}{k}}$ and hence is not affected by the acceleration of the frame of reference.

39 Total energy of the particle performing simple harmonic motion is, $E = K + U = K_{max} + U_{min}$. K is always positive, while U could be positive, negative or zero. If U_{min} is negative and its value is greater than K_{max}, then E would be negative.

40 When the hoop oscillates in its plane, moment of inertia is $I_1 = mR^2 + mR^2$ i.e., $I_1 = 2mR^2$.

While when the hoop oscillates in a direction perpendicular to the plane of the hoop, moment of inertia is

$$I_2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$$

The time period of physical pendulum,

$$T = 2\pi\sqrt{\frac{I}{mgd}}.$$

Here, d is a same in both the cases.